NAME: KEY

APPLIED ALGEBRA 2

FINAL EXAM STUDY GUIDE CHAPTERS 6, 7, 8, 9, & 10

CHAPTER 6:

6.2-> Identifying Polynomial Functions

Decide whether the function is a polynomial function. If it is, write the function in standard form, state the degree, type and leading coefficient.

1. $f(x) = 7x - 4x^2 - 12 + 3x^3$

STD → $f(x) = 3x^3 - 4x^2 + 7x - 12$
Degree → 3
Type → Cubic
Leading Coeff → 5

2. $f(x) = 1 + 3x - \frac{1}{2}x^{-2}$

Not a polynomial Function

Use direct substitution to evaluate the polynomial function for the given value of x.

3. $f(x) = 4x^2 + 2x^3 - x + 7,\ for\ x = -2$

$f(-2) = 4(-2)^2 + 2(-2)^3 - 2(-2) + 7$
$f(-2) = 4(4) + 2(-8) + 2 + 7$
$f(-2) = 16 - 16 + 2 + 7$
$f(-2) = 9$

4. $f(x) = x^3 + 5x^2 + 4x + 6,\ for\ x = 2$

$f(2) = 2^3 + 5(2)^2 + 4(2) + 6$
$f(2) = 8 + 20 + 8 + 6$
$f(2) = 42$

5. $f(x) = x^3 + x^5 + 3,\ for\ x = -1$

$f(-1) = (-1)^3 + (-1)^5 + 3$
$f(-1) = -1 + (-1) + 3$
$f(-1) = 1$

KEY

6.3 Part One-> Adding and Subtracting Polynomials

Find each sum or difference.

6. $(5x^2 + 2x + 1) + (4x^2 + 3x - 8)$

$9x^2 + 5x - 7$

7. $(4x^2 + x + 6) + (7x + 10)$

$4x^2 + 8x + 16$

8. $(5x^2 - 6x - 1) - (4x^2 - 2x + 1)$

$5x^2 - 6x - 1 - 4x^2 + 2x - 1$

$\boxed{x^2 - 4x - 2}$

9. $(-8x^2 + x + 5) - (2x^2 - 3)$

$-8x^2 + x + 5 - 2x^2 + 3$

$\boxed{-10x^2 + x + 8}$

6.1-> Properties of Exponents

Evaluate each expression by applying exponent rules.

10. $(2^3)^2$

$2^6 =$

64

11. $\left(\frac{2}{3}\right)^3$

$\frac{2^3}{3^3} = \frac{8}{27}$

12. $\left(\frac{1}{5}\right)^{-2}$

$\left(\frac{5}{1}\right)^2 = \frac{5^2}{1^2} =$

$\frac{25}{1} = 25$

13. $(2x^2)^5$

$2^5 x^{10}$

$32x^{10}$

14. $\frac{y^{-2}}{y^3}$

$\frac{1}{y^2 y^3} = \boxed{\frac{1}{y^5}}$

15. $\frac{x^7 y^4}{x^{-1} y^{-2}}$

$x^{7-(-1)} y^{4-(-2)}$

$\boxed{x^8 y^6}$

16. $\frac{x^4 y^{10}}{xy^3}$

$x^{4-1} y^{10-3}$

$\boxed{x^3 y^7}$

17. $\frac{(3x^{-2}y^4)^2}{(-2x^5y^3)^{-2}}$

$\frac{3^2 x^{-2(2)} y^{4(2)}}{(-2)^{-2} x^{5(-2)} y^{3(-2)}} = \frac{9x^{-4} y^8}{(-2)^{-2} x^{-10} y^{-6}} =$

$9(-2)^2 x^{-4-(-10)} y^{8-(-6)} = \boxed{36x^6 y^{14}}$

KEY

6.3 Part Two-> Multiplying Polynomials & FOIL Method

Find the product of the polynomials.

18. $(x-2)(x^2+x-2)$

$x^3-x^2-2x-2x^2-2x+4$

x^3-x^2-4x+4

19. $(2x+7)(2x-7)$

$4x^2-14x+14x-49$

$4x^2-49$

20. $(x-3)^2$

$(x-3)(x-3)$

$x^2-3x-3x+9$

x^2-6x+9

21. $(x-1)^3$

$(x-1)(x-1)(x-1)$

$(x-1)(x^2-x-x+1)$

$(x-1)(x^2-2x+1)$

x^3-3x^2+3x-1

6.4 -> Factoring Polynomials

Sum of Cubes-> $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of Cubes-> $a^3-b^3=(a+b)(a^2+ab+b^2)$

Factor each polynomial using the sum or difference of cubes.

22. x^3+125 $a=x$ $b=5$

$(x+5)(x^2-5x+25)$

23. x^3-343 $a=x$ $b=7$

$(x-7)(x^2+7x+49)$

24. $64x^3-1$

$a=4x$

$b=1$

$(4x-1)(16x^2+4x+1)$

25. $8x^3+27$

$a=2x$

$b=3$

$(2x+3)(4x^2+6x+9)$

KEY

Factor each polynomial by grouping.

26. $x^3 - x^2 - 9x + 9$

$x^2(x-1) - 9(x-1)$

$(x^2-9)(x-1)$

$\boxed{(x+3)(x-3)(x-1)}$

27. $x^3 - x + 5x^2 - 5$

$x(x^2-1) + 5(x^2-1)$

$(x+5)(x^2-1)$

$\boxed{(x+5)(x+1)(x-1)}$

Solve each equation.

28. $3x^2 = 9x$

$3x^2 - 9x = 0$

$3x(x-3) = 0$

$\frac{3x}{3} = \frac{0}{3}$ OR $x - 3 = 0$
$+3 \quad +3$

$x = 0$ OR $x = 3$

29. $x^2 = 2x + 15$

$x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$x + 3 = 0$ OR $x - 5 = 0$
$-3 \quad -3 \qquad +5 \quad +5$

$x = -3$ OR $x = 5$

CHAPTER 7:

7.1-> Finding nth Roots

Find the indicated real nth root(s) of a.

30. n = 2, a = 16

$\sqrt[2]{16} =$

± 4

31. n = 3, a = -1

$\sqrt[3]{-1} =$

-1

32. n = 4, a = -16

$\sqrt[4]{-16}$

$\boxed{\text{undefined}}$

KEY

Evaluate each expression.

33. $27^{2/3}$

$(\sqrt[3]{27})^2 = 3^2 = 9$

9 3

3 3

34. $81^{-3/4}$

$\frac{1}{81^{3/4}} = \frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \frac{1}{27}$

9 9

3 3 3 3

35. $\sqrt[3]{-8} = -2$

+4 -2

-2 -2

36. $(\sqrt{36})^3 = 6^3 = 216$

6 6

Evaluate each expression. Leave you answer in simplest radical form.

37. $\sqrt[4]{252}$

$252^{1/4} =$

3.98

38. $\sqrt[3]{2{,}111}$

$2111^{1/3} =$

12.82

39. $(\sqrt[3]{56})^4$

$56^{4/3} =$

214.25

Solve the equation. Leave your answer in simplest radical form.

40. $\frac{2x^3}{2} = \frac{92}{2}$

$\sqrt[3]{x^3} = \sqrt[3]{46}$

2 23

$x = \sqrt[3]{46}$

41. $\sqrt[5]{(x-1)^5} = \sqrt[5]{12}$

$x - 1 = \sqrt[5]{12}$

$x = 1 + \sqrt[5]{12}$

KEY

7.2-> Using Properties of Radicals

Use the properties of rational exponents to simplify the expression.

42. $y^{-2/3}$

$\frac{1}{y^{2/3}} = \boxed{\frac{1}{\sqrt[3]{y^2}}}$

43. $\frac{y^{2/3}}{y^{1/3}}$ $y^{2/3-1/3} = y^{1/3}$ OR $\boxed{\sqrt[3]{y}}$

44. $\frac{1}{64^{-1/3}}$

$64^{1/3} = \sqrt[3]{64} = 2 \cdot 2 = 4$

8 8

4 2 4 2

45. $z^{2/3} \cdot z^{1/2}$

$z^{2/3+1/2} =$

$z^{4/6+3/6} = z^{7/6} = \sqrt[6]{z^7} =$

$\boxed{z \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z}$

$\boxed{z\sqrt[6]{z}}$

Use the properties of radicals to simplify the expression.

46. $\sqrt[3]{16} \cdot \sqrt[3]{4}$

4 4

2 2 2 2 2 2

$\cancel{2\sqrt[3]{2}}$ $2 \cdot 2 =$

4

47. $\sqrt{6} \cdot \sqrt{6} =$

$\sqrt{36} =$

6

48. $\frac{\sqrt[4]{32}}{\sqrt[4]{2}} = \sqrt[4]{16}$

4 4

2 2 2 2

2

49. $\frac{\sqrt[3]{250}}{\sqrt[3]{2}} = \sqrt[3]{125} = 5$

25 5

5 5

KEY

Write the expression in simplest form. Assume all variables are positive.

★50. $\sqrt[4]{256x^8y}$

16 16

4 4 4 4

2 2 2 2 2 2 2 2

$\boxed{4x^2\sqrt[4]{y}}$

51. $\sqrt{\frac{4x^2y}{9z^2}}$

$\boxed{\frac{2x\sqrt{y}}{3z}}$

Perform the indicated operation. Assume all variables are positive.

52. $2\sqrt[5]{3} - \sqrt[5]{3}$

$\sqrt[5]{3}$

53. $7(2^{1/8}) + 4(2^{1/8})$

$11(2^{1/8})$

54. $4\sqrt{x} + 2\sqrt{x}$

$6\sqrt{x}$

7.3-> Operations with Functions

Let $f(x) = 2 - x$ and $g(x) = 3x$.

Add or subtract the following functions.

55. $f(x) + g(x)$

$(2-x)+3x$

$\boxed{2x+2}$

56. $f(x) - g(x)$

$(2-x)-3x$

$\boxed{-4x+2}$

KEY

Multiply or divide the following functions.

57. $f \cdot g;\ f(x) = x^{1/2}, g(x) = 3x^3$

$x^{1/2} \cdot 3x^3 = 3x^{1/2 + 6/2} = \boxed{3x^{7/2}}$

58. $\frac{f}{g};\ f(x) = 4x^{2/3},\ g(x) = 2x$

$\frac{4x^{2/3}}{2x^1} = \frac{4x^{2/3}}{2x^{3/3}} = 2x^{-1/3} = \boxed{\frac{2}{x^{1/3}} \text{ OR } \frac{2}{\sqrt[3]{x}}}$

Let $f(x) = 2x^{-1}$ and $g(x) = x - 2$.

Perform the indicated operation.

59. $f(g(x))$

$2(x-2)^{-1} = \boxed{\frac{2}{x-2}}$

60. $g(g(x))$

$(x-2)-2 = \boxed{x-4}$

7.4-> Inverse Functions

Find an equation for the inverse relation.

60. $y = 4x + 8$

$x = 4y + 8$

$4y = x - 8$

$\boxed{y = \frac{1}{4}x - 2}$

61. $y = -3x + 12$

$x = -3y + 12$

$-3y = x - 12$

$\boxed{y = -\frac{1}{3}x + 4}$

KEY

Solve the equation. Check your solution.

66. $(\sqrt{4+3x})^2 = (10)^2$

$4 + 3x = 100$

$-4 \quad -4$

$\frac{3x}{3} = \frac{96}{3}$

$\boxed{x = 32}$

67. $(\sqrt[3]{4x-1})^3 = (3)^3$

$4x - 1 = 27$

$\frac{4x}{4} = \frac{28}{4}$

$\boxed{x = 7}$

8.1

Exponential Growth and Decay Word Problems. KEY

Compound Interest Formula-> $y = a(1 + \frac{r}{n})^{t \cdot n}$, *where* $t = years, a =$ *initial amount,* $r =$ *rate as a decimal, and* $n =$ *number of times compounded.*

Exponential Growth Formula-> $y = a(1 + r)^t$, *where* $t = years, a =$ *initial amount,* $r = \%$ *rate as a decimal*

Exponential Decay Formula-> $y = a(1 - r)^t$, *where* $t = years, a =$ *initial amount,* $r = \%$ *rate as a decimal*

90. A customer purchases a television set for $800 using a credit card. The interest is charged on any unpaid balance at a rate of 18% per year compounded monthly. If the customer makes no payment for one year, how much is owed at the end of the year?

Compound Interest → $y = a(1 + \frac{r}{n})^{t \cdot n}$

$a = 800$

$r = 18\% = .18$

$t = 1$

$n = 12$

$y = 800(1 + \frac{.18}{12})^{1 \cdot 12}$

$\boxed{y = \$956.49}$

KEY

7.5-> Graphing Square Root Functions

$y = a\sqrt{x-h} + k$; Vertex = (h, k)

Graph the square root function.

What does "h" do?

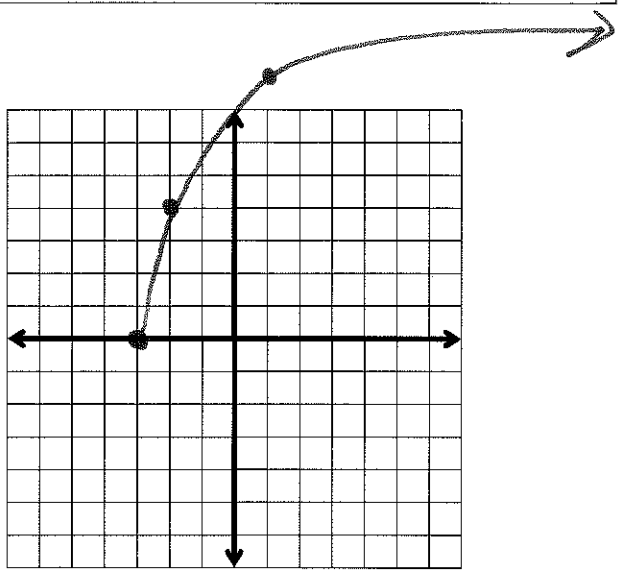
62. $y = 4\sqrt{x+3}$

a=4

h= -3 (add to x)
K=0 (add to y)

Coordinate Point	Multiply y (a)	Add x & y
(0, 0) •	(0,0)	(-3,0)
(1, 1)	(1,4)	(-2,4)
(4, 2)	(4,8)	(1,8)

Graph These

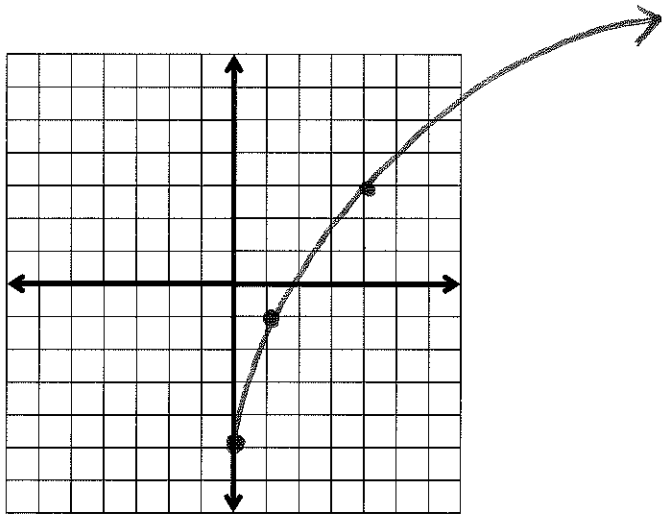


63. $y = 4\sqrt{x} - 5$

a=4

h=0 (add to x)
K=-5 (add to y)

Coordinate Point	Multiply y (a)	Add x & y
(0, 0) •	(0,0)	(0,-5)
(1, 1)	(1,4)	(1,-1)
(4, 2)	(4,8)	(4,3)



7.6-> Radical Equations

Solve the equation. Check your solution.

64. $\sqrt[3]{x} + 2 = 0$

$-2 \quad -2$

$(\sqrt[3]{x})^3 = (-2)^3$

$\boxed{x = -8}$

65. $(\sqrt[4]{x})^4 = 3^4$

$\boxed{x = 81}$

KEY

69. A house was purchased for $90,000 in 1995. If the value of the home increases 5% per year, what will it be worth in the year 2020?

Exponential Growth → $y = a(1+r)^t$

$a = 90{,}000$

$r = 5\% = .05$

$t = 2020 - 1995 = 25$

$y = 90{,}000(1+.05)^{25}$

$\boxed{y = \$304{,}771.94}$

Exponential Decay Word Problems.

Exponential Decay Formula-> $y = a(1-r)^t, where\ t = years, a = initial\ amount, r = \%\ rate\ as\ a\ decimal$

70. The power output is given by the equations $P = 50e^{-\frac{t}{50}}$, where P is the power in watts and t is the time in days.

a. Find the power available after $\underset{t}{\underline{100}}$ days.

$P = 50e^{-\frac{100}{50}}$

$P = 50e^{-2}$

b. Ten watts of power are required to operate the equipment in the satellite. Can the satellite still operate on day 404?

KEY

71. A piece of machinery valued at $250,000 depreciates at 12% per year. What will the value of the machinery be after 8 years?

Exponential Decay $\rightarrow y = a(1-r)^t$

$a = \$250{,}000$

$r = 12\% = .12$

$t = 8$

$y = 250{,}000\,(1-.12)^8$

$\boxed{y = \$89{,}908.63}$

8.3-> Operations with "e"

Simplify each expression. Remember to treat "e" just like a variable.

72. $e^4 \cdot e^6$

e^{4+6}

$\boxed{e^{10}}$

73. $(2e^3)^3$

$2^3 e^{3(3)}$

$\boxed{8e^9}$

74. $\frac{10e^2}{2e^4}$

$5e^{2-4}$

$5e^{-2}$

$\boxed{\frac{5}{e^2}}$

8.4-> Logarithmic Functions

$$\log_b y \leftrightarrow b^x = y$$

Rewrite each equation in exponential form.

75. $log_4\, 64 = 4$

$4^4 = 64$

77. $log_2 \frac{1}{8} = -3$

$2^{-3} = \frac{1}{8}$

* Since $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

78. $log_8\, 8 = 1$

$8^1 = 8$

79. $log_{1/3}\, 3 = -1$

$\frac{1}{3}^{-1} = 3$

KEY

8.5-> Properties of Logarithms

Product Property-> $log_b\, uv = log_b\, u + log_b\, v$

Quotient Property-> $log_b\, \frac{u}{v} = log_b\, u - log_b\, v$

Power Property-> $log_b\, u^n = n\, log_b\, u$

Change-of-Base Formula-> $log_c\, u = \frac{log_b u}{log_b\, c}$ OR $log_c\, u = \frac{\log u}{\log c}$ OR $log_c\, u = \frac{\ln u}{\ln c}$

**Note-> b, c, and u must be positive numbers. $b \neq 1$ and $c \neq 1$

Expand the expression.

80. $\log 9x$

$\log 9 + \log x$

81. $log_2 6x^3$

$3 \log_2 6x$

82. $log_3 \frac{4x}{5}$

$\log_3 4x - \log_3 5$

83. $\ln \frac{2x^2}{y}$

$\ln 2x^2 - \ln y$

$\ln 2 + \ln x^2 - \ln y$

$\boxed{\ln 2 + \ln x + \ln x - \ln y}$

Condense the expression.

84. $log_4\, 12 + log_4\, 5$

$\log_4 12(5) =$

$\boxed{\log_4 60}$

85. $\ln 3 + \ln 6 - \ln 9$

$\ln 3(6) - \ln 9$

$\ln 18 - \ln 9$

$\ln \frac{18}{9} = \boxed{\ln 2}$

KEY

CHAPTER 9:

Simplify the rational expressions.

86. $\frac{x^2-2x+1}{x^2-1} = \frac{(x-1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} =$

$\boxed{\frac{(x-1)}{(x+1)}}$

87. $\frac{x^2-2x-15}{x^2-4x-5} = \frac{(x+3)\cancel{(x-5)}}{(x+1)\cancel{(x-5)}} =$

$\boxed{\frac{(x+3)}{(x+1)}}$

Multiply the rational expressions.

88. $\frac{\cancel{4}^{1}y^2}{\cancel{9}_{1}x} \cdot \frac{\cancel{27}^{3}}{\cancel{16}_{4}xy^2} = \frac{3y^{\cancel{2}}}{4x^2\cancel{y^2}} =$

$\boxed{\frac{3}{4x^2}}$

89. $\frac{x^2-2x}{x^2+2x+1} \cdot \frac{x^2+4x+3}{x^2+3x}$

$\frac{\cancel{x}(x-2)}{(x+1)\cancel{(x+1)}} \cdot \frac{\cancel{(x+3)}\cancel{(x+1)}}{\cancel{x}\cancel{(x+3)}} =$

$\boxed{\frac{(x-2)}{(x+1)}}$

Divide the rational expressions.

90. $\frac{x^2-3x+2}{25x} \div \frac{x-1}{5x^2} = \frac{\cancel{(x-1)}(x-2)}{\cancel{25x}_{5}} \cdot \frac{\cancel{5}x^{\cancel{2}}}{\cancel{(x-1)}} = \boxed{\frac{x(x-2)}{5}}$

CHAPTER 10:

Distance Formula → $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Find the distance between the two points.

90. $(3, -2), (-1, 4)$ (x_1, y_1 x_2, y_2)

$d=\sqrt{(-1-3)^2+(4-(-2))^2}$

$d=\sqrt{(-4)^2+6^2}$

$d=\sqrt{16+36}$

$d=\sqrt{52}$

$\boxed{d=7.21\ldots}$

91. $(6, 1), (2, -5)$ (x_1, y_1 x_2, y_2)

$d=\sqrt{(2-6)^2+(-5-1)^2}$

$d=\sqrt{(-4)^2+(-6)^2}$

$d=\sqrt{16+36}$

$d=\sqrt{52}$

$\boxed{d=7.21\ldots}$

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

KEY

Find the midpoint of the line segment joining the two points.

93. $(-2, -5), (4, 6)$ (x_1, y_1 x_2, y_2)

$\left(\frac{-2+4}{2}, \frac{-5+6}{2}\right)$

$\left(\frac{2}{2}, \frac{1}{2}\right)$

$\boxed{\left(1, \frac{1}{2}\right)}$

94. $(-10, 12), (-6, -14)$ (x_1, y_1 x_2, y_2)

$\left(\frac{-10+(-6)}{2}, \frac{12+(-14)}{2}\right)$

$\left(\frac{-16}{2}, \frac{-2}{2}\right)$

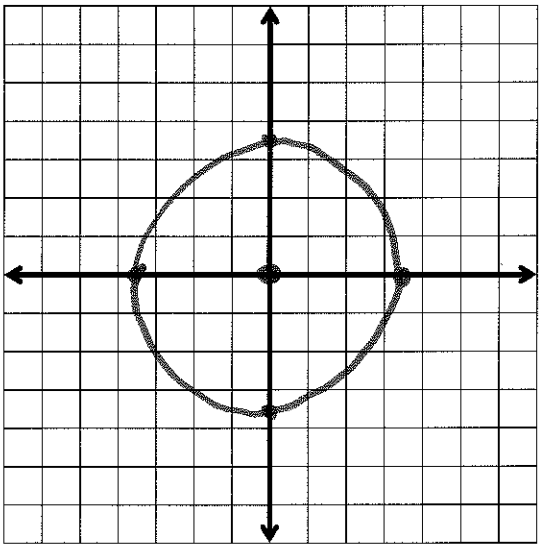
$\boxed{(-8, -1)}$

Graph each circle and give the center and radius.

95. $x^2 + y^2 = 12$

Center: $(0,0)$

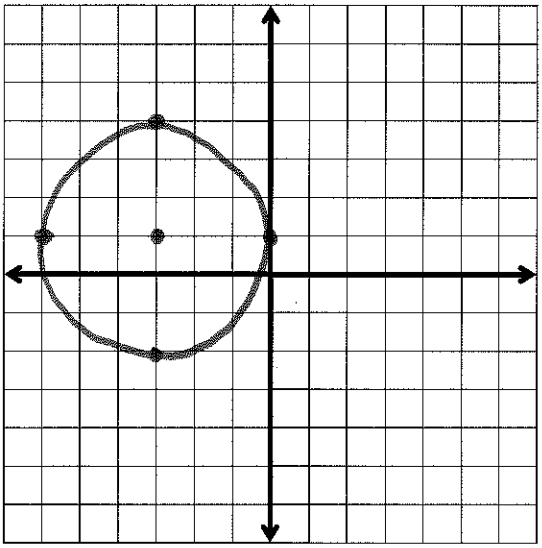
Radius: $\sqrt{12} = 3.5$



96. $(x+3)^2 + (y-1)^2 = 9$

Center: $(-3,1)$

Radius: 3



KEY

Graph the following and identify the major axis, vertices and co-vertices.

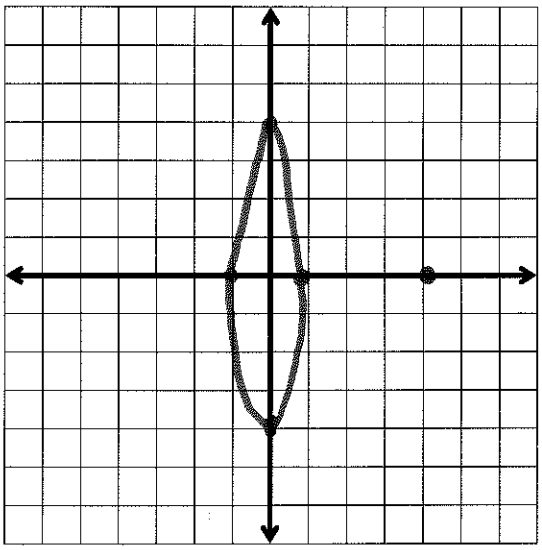
97. $\frac{x^2}{1} + \frac{y^2}{16} = 1$

$b = \sqrt{1} = 1$ ↳ $a = \sqrt{16} = 4$

Major axis: Vertical

Vertices: (0, a) (0, -a) (0, 4) (0, -4)

Co-Vertices: (b, 0) (-b, 0) (1, 0) (-1, 0)



98. $\frac{x^2}{36} + \frac{y^2}{9} = 1$

$a = \sqrt{36} = 6$ ↳ $b = \sqrt{9} = 3$

Major axis: Horizontal

Vertices: (a, 0) (-a, 0) (6, 0) (-6, 0)

Co-Vertices: (0, b) (0, -b) (0, 3) (0, -3)