Name: KEY

Topic: 8.6 Solving Exponential and Logarithmic Equations – Day 2

Summary:

Use this method when:

① Only 1 "x" exponent.

② Can't create like bases.

*Remember:

$\log_b A^n = n \cdot \log_b A$

(Exponent becomes coefficient in front).

Change-of -Base Formula

Let u, b and c be positive numbers with $b \neq 1$ and $c \neq 1$. Then:

$$\log_c u = \frac{\log u}{\log c} \quad \text{AND} \quad \log_c u = \frac{\ln u}{\ln c}$$

Solve using logarithms

Example 1: $3^{-x} - 6 = 9$

$+6 \quad +6$

$3^{-x} = 15$

*Now take log of both sides.

$\log 3^{-x} = \log 15$

$-x \log 3 = \log 15$

$-x = \dfrac{\log 15}{\log 3}$

$\dfrac{-x}{-1} \approx \dfrac{2.465}{-1}$

$\boxed{x \approx -2.465}$

*NOTE:

Get any #^x by itself first.

Example 2: $4^x = 15$

*Take log of both sides.

$\log 4^x = \log 15$

$x \cdot \log 4 = \log 15$

$x = \dfrac{\log 15}{\log 4}$

$\boxed{x \approx 1.953}$

8.6
Day 2

Practice: Solve the equation using log

1. $5^x = 8$

*Take log of both sides

$\log 5^x = \log 8$

$x \cdot \log 5 = \log 8$

$x = \frac{\log 8}{\log 5}$

$x \approx 1.292$

2. $e^{-x} = 5$

* Take $\log_e$ of both sides

$\log_e e^{-x} = \log_e 5$

$-x \cdot \log_e e = \log_e 5$

$-x = \log_e 5$

* $\log_e = \ln$

$-x = \ln 5$

$x = -(\ln 5)$

$x \approx -1.609$

*How to cancel base "e":

Take ln, or $\log_e$, of both sides. Then cancel "e" bases.

3. $2^x + 1 = 5$

$-1 \quad -1$

$2^x = 4$

*Take log of both sides

$\log 2^x = \log 4$

$x \cdot \log 2 = \log 4$

$x = \frac{\log 4}{\log 2}$

$x \approx 2$

4. $10^{2x-6} = 146$

*Take log of both sides

$\log 10^{2x-6} = \log 146$

$(2x-6) \log 10 = \log 146$

$2x - 6 = \frac{\log 146}{\log 10}$

$2x - 6 = 2.164$

$2x = 8.164$

$x \approx 4.082$

5. $9 - 4e^x = 5$

$-9 \quad -9$

$-4e^x = -4$

$e^x = 1$

*Take $\log_e$ of both sides.

$\log_e e^x = \log_e 1$

$x = \log_e 1$

* $\log_e = \ln$

$x = \ln 1$

$x \approx 0$

6. $\frac{1}{2} e^{-2x} = 6$

$(2) \quad (2)$

$e^{-2x} = 12$

* take $\log_e$ of both sides

$\log_e e^{-2x} = \log_e 12$

$-2x = \log_e 12$

* $\log_e = \ln$

$-2x = \ln 12$

$x = \frac{\ln 12}{-2}$

$x \approx -1.242$

8.6 Day 2

Solving a Logarithmic Equation

Example 3: $\ln(2x+3) = \ln(5x-6)$

*ln cancels out on both sides.

$2x+3=5x-6$

$-2x \quad -2x$

$3=3x-6$

$+6 \quad +6$

$9=3x$

$\frac{9x}{3}=\frac{9}{3}$

$x=3$

Example 4:

$\log_4(x+3) = 2$

*Remember: $\log_a b = x \longleftrightarrow a^x = b$

$4^2 = x+3$

$16 = x+3$

$-3 \quad -3$

$x=13$

$\ln x = 5$

*ln = $\log_e$

$\log_e x = 5$

$x = e^5$

$x \approx 148.413$

TRY:

$\log_4(x+3) = \log_4(8x+17)$

* $\log_4$ cancels out on both sides.

$x+3=8x+17$

$-x \quad -x$

$3=7x+17$

$-17 \quad -17$

$\frac{-14}{7}=\frac{7x}{7}$

$x=-2$

$\frac{4\log_3 3x}{4} = \frac{20}{4}$

$\log_3 3x = 5$

*Remember: $\log_a b = x \longleftrightarrow a^x = b$

$3^5 = 3x$

$\frac{243}{3}=\frac{3x}{3}$

$x=81$

8.6 Day 2

Solve the following equations.

7. $\ln(4-x) = \ln(4x-11)$

* ln cancels out on both sides

$4 - x = 4x - 11$
$+x \quad +x$
$4 = 5x - 11$
$+11 \quad +11$
$15 = 5x$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

8. $\log_8(x-5) = \frac{2}{3}$

*Remember: $\log_a b = x \longleftrightarrow a^x = b$

$8^{2/3} = x - 5$

$\sqrt[3]{8^2} = x - 5$

$2^2 = x - 5$

$4 = x - 5$

$x = 9$

9. $3\log_5(x+2) = 6$

$\frac{3\log_5(x+2)}{3} = \frac{6}{3}$

$\log_5(x+2) = 2$

*Remember: $\log_a b = x \longleftrightarrow a^x = b$

$5^2 = x + 2$

$25 = x + 2$
$-2 \quad -2$

$x = 23$

10. $4\ln 2x = 5$

$\frac{4\ln 2x}{4} = \frac{5}{4}$

* ln = log$_e$

$\log_e 2x = \frac{5}{4}$

$e^{5/4} = 2x$

$x = \frac{e^{5/4}}{2}$

$x \approx 1.745$

11. $\log_2(x-1) = \log_2(2x+1)$

* $\log_2$ cancels on both sides.

$x - 1 = 2x + 1$
$-x \quad -x$
$-1 = x + 1$
$-1 \quad -1$

$x = -2$

12. $-5 + 2\ln x = 5$
$+5 \quad +5$

$\frac{2\ln x}{2} = \frac{10}{2}$

$\ln x = 5$

* ln = log$_e$

$\log_e x = 5$

$x = e^5$

$x \approx 148.413$

13. $\ln(4x-9) = \ln x$

* ln cancels out on both sides

$4x - 9 = x$
$-4x \quad -4x$

$\frac{-9}{-3} = \frac{-3x}{-3}$

$x = 3$

14. $2 + \log_3 2x = -3$
$-2 \quad -2$

$\log_3 2x = -5$

*Remember: $\log_a b = x \longleftrightarrow a^x = b$

$3^{-5} = 2x$

$\frac{1}{3^5} = 2x$

$\frac{1}{243} = 2x$

$\frac{2x}{2} = \frac{\frac{1}{243}}{2}$

$x = \frac{1}{486}$