Name: KEY

Topic: 8.6 Solving Exponential and Logarithmic Equations

Summary: Part 1

Change-of -Base Formula

Let u, b and c be positive numbers with $b \neq 1$ and $c \neq 1$. Then:

$$\log_c u = \frac{\log u}{\log c} \quad \text{AND} \quad \log_c u = \frac{\ln u}{\ln c}$$

Examples:

1. $\log_2 4 = \frac{\log 4}{\log 2} \approx \frac{.602}{.301} \approx 2$

   $*2^2=4$

2. $\log_9 729 = \frac{\log 729}{\log 9} \approx \frac{2.863}{0.954}$ $3.001 \approx 3$

   $*9^3=729$

3. $\log_2 5 = \frac{\log 5}{\log 2} \approx \frac{.699}{.301}$ 2.322

4. $\log_7 10 = \frac{\log 10}{\log 7} \approx \frac{1}{0.845} \approx 1.183$

5. $\log_6 200 = \frac{\log 200}{\log 6} \approx \frac{2.301}{0.778} \approx 2.958$

6. $\log_5 \frac{1}{2} = \frac{\log \frac{1}{2}}{\log 5} \approx \frac{-0.301}{.699} \approx -0.431$

7. $\log_4 1{,}235 = \frac{\log 1235}{\log 4} \approx \frac{3.09}{0.602} \approx 5.13$

8. $\log_3 17 = \frac{\log 17}{\log 3} \approx \frac{1.230}{0.477} \approx 2.579$

8.6

Part 1

Solving equations by equating exponents

Example: solve $2^{4x} = 32^{x-1}$

$2^5 = 32$

$2^{4x} = 2^{5(x-1)}$

$4x = 5(x-1)$

$4x = 5x - 5$

$-x = -5$

$\boxed{x = 5}$

9. $4^x = 4^{2x+1}$

$x = \cancel{2x} + 1$

$-2x \quad -\cancel{2x}$

$-x = 1$

$\boxed{x = -1}$

10. $3^{2x} = 3^{x-5}$

$2x = \cancel{x} - 5$

$-x \quad -\cancel{x}$

$\boxed{x = -5}$

11. $e^{3x} = e^{2x+7}$

$3x = \cancel{2x} + 7$

$-2x \quad -\cancel{2x}$

$\boxed{x = 7}$

12. $e^{2x-1} = e^{3-x}$

$2x - 1 = 3 \cancel{-x}$

$+x \quad \cancel{+x}$

$3x \cancel{-1} = 3$

$\cancel{+1} \quad +1$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{4}{3}$

$\boxed{x = \frac{4}{3}}$

13. $9^{x+1} = 3^{3x-3}$

$3^2 = 9$

$3^{2(x+1)} = 3^{3x-3}$

$\boxed{x = 5}$

$2(x+1) = 3x - 3$

$\cancel{2x} + 2 = 3x - 3$

$-\cancel{2x} \quad -2x$

$2 = x - 3$

14. $4^{x+1} = 16$

$4^2 = 16$

$4^{x+1} = 4^2$

$x + 1 = 2$

$\boxed{x = 1}$