Name: KEY

Topic: 8.4 Logarithmic Functions

Summary:

Let b and y be positive numbers, $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x, \text{ then } b^x = y$$

Logarithm Form

Exponential Form

* Read as "Log base b of y"

Rewrite the equation in exponential form.

1. $\log_2 16 = 4$

$2^4 = 16$

2. $\log_3 27 = 3$

$3^3 = 27$

~~3. $\log_3 27 = 3$~~

Duplicate

4. $\log_5 \frac{1}{5} = -1$

$5^{-1} = \frac{1}{5}$

* Negative Exponent Rule

5. $\log_2 1 = 0$

$2^0 = 1$

* Zero Exponent Rule

6. $\log_3 \frac{1}{27} = -3$

$3^{-3} = \frac{1}{27}$

* Negative Exponent Rule

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

Write in logarithmic form:

1. $5^3 = 125$

$\log_5 125 = 3$

2. $8^{-2} = \frac{1}{64}$

$\log_8 \frac{1}{64} = -2$

*You can also use "Change of Base" property of logarithms. $\log_a B = \frac{\log B}{\log a}$

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* Rewrite in Exponential Form, then "guess and Check" to Solve for x.

Evaluate the expression without using a calculator.

7. $\log_2 4 = x$

$2^x = 4$

$x = 2$

Change of Base: $\frac{\log 4}{\log 2}$

8. $\log_2 32 = x$

$2^x = 32$

$x = 5$

Change of Base: $\frac{\log 32}{\log 2}$

9. $\log_8 2 = x$

Change of Base: $\frac{\log 2}{\log 8}$

$8^x = 2$

$2 \cdot 4$ $\quad \sqrt[3]{8} = 2$

$2 \cdot 2$ $\quad \therefore 8^{\frac{1}{3}} = 2$

$x = \frac{1}{3}$

If your answer is smaller than your base, think $\sqrt{\ }$, $\sqrt[3]{\ }$, etc.

10. $\log_5 5^{2/3} = x$

$5^x = 5^{2/3}$

$x = 2/3$

Change of Base: $\frac{\log 5^{2/3}}{\log 5}$

11. $\log_2 \frac{1}{2} = x$

$2^x = \frac{1}{2}$

$x = -1$

Change of Base: $\frac{\log \frac{1}{2}}{\log 2}$

12. $\log_6 1 = x$

$6^x = 1$

$x = 0$

Change of Base: $\frac{\log 1}{\log 6}$

A logarithm with a base e is called the natural logarithm, and is written:

$\log_e x =$ ln x

A logarithm with a base of 10 is called the common logarithm, and is written:

$\log_{10} x =$ log x

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*Just plug in to calculator.

Evaluate the expression using a calculator. Round to three decimal places.

13. $\log 3.72$

0.571

14. $\ln 0.23$

-1.470

15. $\ln\sqrt{3}$

0.549

*If the base of a power is the same as the base of the log, then the bases will cancel out.

Simplify:

16. $\cancel{4}^{\log_{\cancel{4}} x}$

x

17. $\log_5 125^x$

$*5^3=125$

$\log_{\cancel{5}} \cancel{5}^{3x}$

$3x$

18. $10^{\log x}$

$\cancel{10}^{\log_{\cancel{10}} x}$

x

$f(x)=y$

$\ln=\log_e$

Find the inverse of the function. *Switch x and y.

19. $f(x)=\log_3 x$

$y=\log_3 x$

Inverse:

$x=\log_3 y$

$3^x=y$

$\boxed{y=3^x}$

20. $f(x)=\ln x$

$y=\log_e x$

Inverse:

$x=\log_e y$

$e^x=y$

$\boxed{y=e^x}$

21. $f(x)=\log_{1/3} x$

$y=\log_{\frac{1}{3}} x$

Inverse:

$x=\log_{\frac{1}{3}} y$

$\frac{1}{3}^x=y$

$\boxed{y=\frac{1}{3}^x}$

22. $f(x)=\log 2x$

$y=\log_{10} 2x$

Inverse:

$x=\log_{10} 2y$

$\frac{10^x}{2}=\frac{\cancel{2}y}{\cancel{2}}$

$\boxed{y=\frac{10^x}{2}}$

23. $f(x)=\log_6(2x)$

$y=\log_6(2x)$

Inverse:

$x=\log_6(2y)$

$\frac{6^x}{2}=\frac{\cancel{2}y}{\cancel{2}}$

$\boxed{y=\frac{6^x}{2}}$

24. $f(x)=\log_4 16x$

$y=\log_4 16x$

Inverse:

$x=\log_4 16y$

$\frac{4^x}{16}=\frac{\cancel{16}y}{\cancel{16}}$

$\boxed{y=\frac{4^x}{16}}$