

Name: KEY

Topic: 8.1: Exponential Growth

Date: _____ Block: _____

Summary:

Exponential Growth Function

$$y = ab^x$$

$$a > \underline{0}$$

$$b > \underline{1}$$

Why do I need to learn this? When is it used?

1. Technology → Facebook Started small
2. Financial Planning
3. Population Studies
4. Travel Times
5. Business / Customer Service (Good Service vs. Bad Service)

General equation

$$y = a(1 + r)^t$$

a

Initial
Amount
* What did
you start
with?

r

Rate (%)
of Increase
* written as
a decimal.
Ex. 6% = 0.06

t

Time
* number of
years

$$y = a(1+r)^t \rightarrow \text{time (years)}$$

\downarrow Initial \downarrow rate

Example 1:

In 1980 about 2,180,000 U.S. workers worked at home. During the next ten years, the number of workers working at home increased 5% per year. Write a model giving the number w (in millions) of workers working at home t years after 1980.

$$a = 2,180,000$$

$$r = 5\% \rightarrow 0.05$$

$$t = ?$$

$$y = 2,180,000(1+0.05)^t$$

From this formula, estimate about how many workers worked from home in 1988.

$$a = 2,180,000$$

$$r = 5\% = .05$$

$$t = 8 \text{ years (1988-1980)}$$

$$y = 2,180,000(1+0.05)^8$$

$$y = 2,180,000(1.05)^8$$

$$y = 3,220,852 \text{ people}$$

Example 2:

In 1990 the cost of tuition at a state university was \$4,300. During the next 8 years, the tuition rose 4% each year. Write a model that gives the tuition y (in dollars) t years after 1990.

$$a = 4300$$

$$r = 4\% = .04$$

$$t = ?$$

$$y = 4300(1+0.04)^t$$

From this formula, estimate what the tuition will be this year.

$$y = 4300(1+0.04)^{20}$$

$$y = 4300(1.04)^{20}$$

$$y = \$9421.83$$

2010

$$t = 2010 - 1990$$

$$t = 20 \text{ years}$$

Principle: initial amount (of money)

Compound Interest: Interest that is paid on the principle
AND on interest (previously earned).

Simple Interest: Interest paid on principle ONLY.

Important note about the %:

- Usually shown as an Annual percent
- Interest is usually compounded more than once per year.

CALCULATING INTEREST

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Annual = 1
Semi-annual = 2
Quarterly = 4
Monthly = 12
Daily = 365

A = amount of money in account
P = principle (initial amount)
r = rate (as a decimal)

n = number of times compounded each year
t = time (years)

Example 3:

You deposit \$1,500 to an account that pays 6% annual interest. Find the balance after 1 year if the interest is compounded

a) annually

$$P = 1500$$

$$r = .06$$

$$t = 1$$

$$n = 1$$

$$A = 1500 \left(1 + \frac{.06}{1} \right)^{1 \cdot 1}$$

$$A = 1500 (1.06)^1$$

$$A = \$1590$$

b) semiannually

$$P = 1500$$

$$r = .06$$

$$t = 1$$

$$n = 2$$

$$A = 1500 \left(1 + \frac{.06}{2} \right)^{2 \cdot 1}$$

$$A = 1500 (1.03)^2$$

$$A = \$1,591.35$$

c) monthly

$$P = 1500$$

$$r = .06$$

$$t = 1$$

$$n = 12$$

$$A = 1500 \left(1 + \frac{.06}{12} \right)^{12 \cdot 1}$$

$$A = 1500 ()^{12}$$

$$A = \$1,592.52$$

Example 4:

You deposit \$2,000 to an account that pays 8% annual interest.

a) find the balance after 2 years if the interest is compounded annually.

$$\begin{aligned} P &= 2000 \\ r &= .08 \\ t &= 2 \\ n &= 1 \end{aligned}$$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 2000 \left(1 + \frac{.08}{1}\right)^{1 \cdot 2}$$

$$A = 2000 (1.08)^2$$

$$A = \$2332.80$$

b) Find the balance after 2 years if the interest is compounded quarterly.

$$\begin{aligned} P &= 2000 \\ r &= .08 \\ t &= 2 \\ n &= 4 \end{aligned}$$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 2000 \left(1 + \frac{.08}{4}\right)^{4 \cdot 2}$$

$$A = 2000 (1 + .02)^8$$

$$A = 2000 (1.02)^8$$

$$A = \$2343.32$$