Name: KEY

Topic: 7.1 nth Roots and Rational Exponents

Summary:

Rewriting radical form in exponent notation and exponent notation in radical form:

$\sqrt[out]{a^{in}}$ is the same as $\left(\sqrt[n]{a}\right)^m \rightarrow a^{\frac{m}{n}} \rightarrow a^{\frac{in}{out}} \rightarrow \sqrt[out]{m^{in}}$

Examples:

1. $\sqrt[3]{15^2}$

$15^{\frac{2}{3}}$

2. $\sqrt[6]{3^2}$

$3^{\frac{2}{6}} = 3^{\frac{1}{3}}$

3. $\sqrt[3]{x^7}$

$x^{\frac{7}{3}}$

4. $4^{\frac{1}{3}}$

$\sqrt[3]{4}$

5. $10^{\frac{3}{4}}$

$\sqrt[4]{10^3}$

6. $y^{\frac{8}{13}}$

$\sqrt[13]{y^8}$

7. $\frac{2x^4}{2} = \frac{162}{2}$

$\sqrt[4]{x^4} = \sqrt[4]{81}$ $x = \pm 3$

$81 = 9 \cdot 9$

$9 = 3 \cdot 3$

8. $z^{\frac{7}{10}}$

$\sqrt[10]{z^7}$

9. $\sqrt[2]{z^5}$

$z^{\frac{5}{2}}$

Finding nth roots

$\sqrt[n]{a^m}$

INDEX (n)
Number needed to "circle/cancel"

BASE (a)
Make a factor tree

EXPONENT (m)
How many of the same factor tree

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Examples: Find the nth root $\sqrt[n]{a}$ for the given values of n and a

n = how many factors needed to Circle/Cancel.

10. n = 5; a = 32

$\sqrt[5]{32} = 2$

8 4
4 2 2 2
2 2

*Note: $2^5 = 32$

11. n = 4; a = 128

$\sqrt[4]{128} = 2\sqrt[4]{2^3} = 2\sqrt[4]{8}$

2 64
8 8
4 2 4 2
2 2 2 2

12. n = 8; a = 2

$\sqrt[8]{2}$

13. n = 3; a = 216

$\sqrt[3]{216} = 2 \cdot 3 = 6$

2 108
2 54
2 2 27
2 9 3
3 3

*Note: $6^3 = 216$

14. $\sqrt[3]{81}$

9 9
3 3 3 3

$3\sqrt[3]{3}$

15. $\sqrt[5]{64}$

8 8
4 2 4 2
2 2 2 2 2

$2\sqrt[5]{2}$

16. $\sqrt[3]{24} = 2\sqrt[3]{3}$

8 3
4 2
2 2 2 3

17. $\sqrt[4]{16} = 2$

4 4
2 2 2 2

*Note: $2^4 = 16$

18. $(1)^{\frac{1}{6}}$

$\sqrt[6]{1} = 1$

19. $(4)^{\frac{3}{5}}$

*same as #15

$\sqrt[5]{4^3} = \sqrt[5]{64} =$

$2\sqrt[5]{2}$

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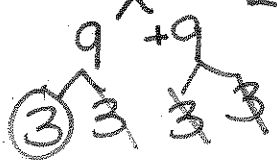
Even/Odd Rules: $\sqrt[\text{index}]{\text{Base}}$

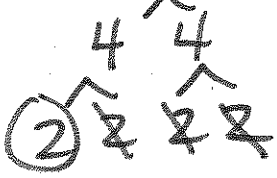
If the index is odd → the nth root will have the same sign as the base

If the index is even → the nth root will be ± (if the base number is negative I will be used)

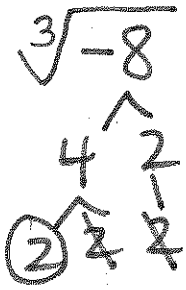
20. $\sqrt[4]{-81} = \boxed{\pm 3i}$



21. $\sqrt[4]{-16} = \boxed{\pm 2i}$

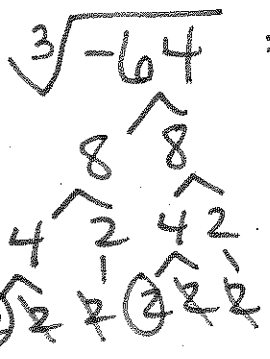


22. $(-8)^{\frac{1}{3}} = \boxed{-2}$



*Keep sign of base → (-8)

23. $(-64)^{\frac{1}{3}}$



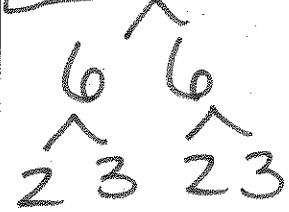
$= -2 \cdot 2 = \boxed{-4}$

*Keep sign of base → (-64)

24. $6^{\frac{2}{3}}$

$\sqrt[3]{6^2} =$

$\boxed{\sqrt[3]{36}}$

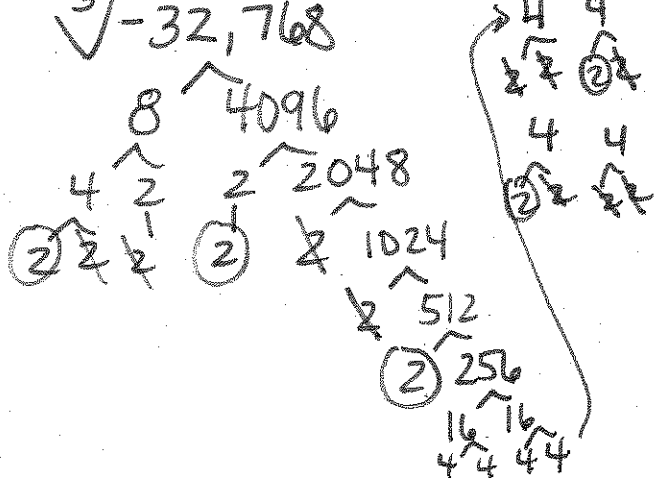


*Cannot factor out a group of 3.

25. $(-8)^{\frac{5}{3}}$

$\sqrt[3]{(-8)^5} = 2^5 =$

$\sqrt[3]{-32,768}$



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Solving equations using nth roots

Evens → ± odds → keep sign

26. $2x^4 = 162$

$\frac{2x^4}{2} = \frac{162}{2}$

$\sqrt[4]{x^4} = \sqrt[4]{81}$

$x = \pm 3$

27. $x^2 - 5 = 139$

$+5 \quad +5$

$\sqrt{x^2} = \sqrt{144}$

$x = \pm 12$

28. $\sqrt[3]{(x-7)^3} = \sqrt[3]{729}$

729 → 3 · 243 → 3 · 81 → 9 · 9 → 3 · 3 · 3 · 3

$x - 7 = 3^2$

$x - 7 = 9$

$+7 \quad +7$

$x = 16$

29. $\frac{5y^4}{5} = \frac{80}{5}$

$\sqrt[4]{y^4} = \sqrt[4]{16}$

$y = \pm 2$

30. $\sqrt[3]{(y-1)^3} = \sqrt[3]{32}$

32 → 8 · 4 → 2 · 2 · 2 · 4

$y + 1 = 2\sqrt[3]{4}$

$-1 \quad -1$

$y = 1 + 2\sqrt[3]{4}$

31. $\sqrt{(x+4)^2} = \sqrt{0}$

$x + 4 = 0$

$-4 \quad -4$

$x = -4$

← *can't do ±0

32. $\frac{-5x^2}{-5} = \frac{-30}{-5}$

$\sqrt{x^2} = \sqrt{6}$

$x = \pm\sqrt{6}$

33. $\sqrt[5]{(x-1)^5} = \sqrt[5]{243}$

$x - 1 = 3$

$+1 \quad +1$

$x = 4$

243 → 3 · 81 → 9 · 9 → 3 · 3 · 3 · 3