sec. 5.6: Describing Number Patterns

inductive reasoning: making conclusions based on patterns you observe

conjecture: a conclusion you reach by inductive reasoning

"look for pa

① Extending Number Patterns: use inductive reasoning to describe each pattern. Then find the next 2 numbers in each pattern.

a) 3, 9, 27, 81, ...
×3

243, 729

b) 9, 15, 21, 27, ...
+6

33, 39

c) 2, -4, 8, -16
× -2

32, -64

sequence: a number pattern

term: each number in a sequence

example: 1, 3, 5, 7, 9, 11, ... sequence

↘ ↓ ↓ ↓ ↓

term(s)

<u>arithmetic sequence</u>: one kind of number sequence formed by <u>adding</u> a fixed number to each previous term. This fixed number is called the <u>common</u> <u>difference</u>.

ex: -4,   5,   14,   23

$$+9 \quad +9 \quad +9$$

↓
common difference

② Find the <u>common difference</u> of each sequence
a) 11, 23, 35, 47, ...   b) 8, 3, -2, -7, ...
   +12   **-5**

* You can use the common difference to write a function rule for the sequence:

$$A(n) = \underline{a + (n-1)d}$$

↓   ↓   ↓   ↘
nth term   1ST   term   common
                term   number   difference

1 EXAMPLE Use inductive reasoning to describe the pattern. Then find the next two numbers in the pattern.

a. 1,   5,   9

+ 4   + 4

The pattern is "add 4 to the previous term." To find the next two numbers, you add 4 to each previous term: $9 + 4 = 13$ and $13 + 4 = 17$.

b. 1,   3,   9

× 3   × 3

The pattern is "multiply the previous term by 3." To find the next two numbers, you multiply each previous term by 3: $9 \times 3 = 27$ and $27 \times 3 = 81$.

c. 1, 9, 25, 49, . . .

The pattern is "square of consecutive odd integers." To find the next two numbers, square the next two consecutive integers: $9^2 = 81$, $11^2 = 121$.

2 EXAMPLE Find the common difference of each arithmetic sequence.

a. 5,   2,   −1,   −4

−3   −3   −3

The common difference is −3.

b. 8,   11,   14,   17

+3   +3   +3

The common difference is 3.

3 EXAMPLE Find the first, fifth, and tenth terms of the sequence that has the rule $A(n) = 15 + (n - 1)(5)$.

first term: $A(1) = 15 + (1 - 1)(5) = 15 + 0(5) = 15$

fifth term: $A(5) = 15 + (5 - 1)(5) = 15 + 4(5) = 35$

tenth term: $A(10) = 15 + (10 - 1)(5) = 15 + 9(5) = 60$

Practice 5-6

Find the common difference of each arithmetic sequence.

1. $10, 16, 22, 28, \ldots$ **2.** $9, 6, 3, 0, \ldots$

3. $-12, -17, -22, -27, \ldots$ **4.** $-11, -8, -5, -2, \ldots$

5. $4, 4\frac{1}{2}, 5, 5\frac{1}{2}, \ldots$ **6.** $7\frac{1}{2}, 7, 6\frac{1}{2}, 6, \ldots$

7. $9, 10.5, 12, 13.5, \ldots$ **8.** $1, -1.5, -4, -6.5, \ldots$

9. $8, 9.1, 10.2, 11.3, \ldots$ **10.** $-9, -8.1, -7.2, -6.3, \ldots$

11. $-3, -0.6, 1.8, 4.2, \ldots$ **12.** $6.2, 4.5, 2.8, 1.1, \ldots$

Find the next two terms in each sequence.

13. $1, 7, 13, 19, \ldots$ **14.** $-8, -5, -2, 1, \ldots$

15. $1, -4, -9, -14, \ldots$ **16.** $\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, \ldots$

17. $2.7, 4, 5.3, 6.6, \ldots$ **18.** $9.8, 0.7, -8.4, -17.5, \ldots$

19. $6\frac{1}{3}, 4\frac{2}{3}, 3, 1\frac{1}{3}, \ldots$ **20.** $2\frac{1}{2}, \frac{3}{4}, -1, -2\frac{3}{4}, \ldots$

Find the fifth, tenth, and hundredth terms of each sequence.

21. $4, 14, 24, 34, \ldots$ **22.** $14, 6, -2, -10, \ldots$

23. $3, 10, 17, 24, \ldots$ **24.** $-19, -22, -25, -28, \ldots$

25. $\frac{1}{4}, -\frac{1}{4}, -\frac{3}{4}, -\frac{5}{4}, \ldots$ **26.** $-1.3, -0.3, 0.7, 1.7, \ldots$

27. $0, 101, 202, 303, \ldots$ **28.** $-1, -100, -199, -298, \ldots$

29. $5, 3.9, 2.8, 1.7, \ldots$ **30.** $-3\frac{1}{2}, -3\frac{3}{4}, -4, -4\frac{1}{4}, \ldots$

Determine whether each sequence is arithmetic. Justify your answer.

31. $0.5, 0.3, 0.1, -0.1, \ldots$ **32.** $-1, 1, -1, 1, \ldots$

33. $3, 6, 12, 24, \ldots$ **34.** $100, 81, 64, 49, \ldots$

35. Renting a backhoe costs a flat fee of $65 plus an additional $35 per hour.

     a. Write the first four terms of a sequence that represents the total cost of renting the backhoe for 1, 2, 3, and 4 hours.

     b. What is the common difference?

     c. What are the 5th, 24th, 48th, and 72nd terms in the sequence?

Reteaching 5-6

Describing Number Patterns

OBJECTIVE: Finding the common difference and writing the next several terms in a sequence	**MATERIALS:** None

When trying to determine the common difference of an arithmetic sequence or find the pattern, it is helpful to attempt to express each term in the sequence as an expression involving the same number.

Example

Find the common difference of the sequence: $4, 7, 10, 13, \ldots$

Let n = the term number in the sequence.
Let $A(n)$ = the value of the nth term of the sequence.

$A(1) = 4$

$A(2) = 7 = 4 + 1(3)$ ⟵ **3 is the common difference.**

$A(3) = 10 = 4 + 6 = 4 + 2(3)$ ⟵ **Notice that the 3 in each expression is multiplied by a number one less than the term number.**

$A(4) = 13 = 4 + 9 = 4 + 3(3)$

$A(n) = 4 + \underbrace{3 + 3 + \ldots + 3}_{n - 1 \text{ terms}} = 4 + (n - 1)3$

The formula for the sequence is $A(n) = 4 + (n - 1)3$.

You could use the formula for the sequence to determine the next several terms simply by substituting a specific term in for n. For example:

Term:

5th term
$A(5) = 4 + (5 - 1)3$
$A(5) = 16$

100th term
$A(100) = 4 + (100 - 1)3$
$A(100) = 301$

Exercises

Find the common difference of each sequence.

1. $2, 9, 16, 23, \ldots$　　　**2.** $5, 1, -3, -7, \ldots$　　　**3.** $-52, -41, -30, -19, \ldots$

Find the next two terms in each sequence.

4. $2, 0, -2, -4, \ldots$　　　**5.** $-4, -1, 2, 5, \ldots$　　　**6.** $-17, -22, -27, -32, \ldots$

Find a formula for the sequence in the exercise indicated and use it to determine the fifth and tenth terms of the sequence.

7. Exercise 1　　　**8.** Exercise 2　　　**9.** Exercise 3