Algebra 1 Notes

8.6

[redacted]-> GEOMETRIC SEQUENCES

Geometric sequence:

Ex: 2, 10, 50, 250

*To find the common ratio, write each term over the term to its left and simplify.

1. Find the common ratio of each sequence:

a. -3, -6, -12, -24, ...

b. 3, -15, 75, -375, ...

c. 750, 150, 30, 6, ...

d. 3, 3/2, $\frac{3}{4}$, 3/8, ...

2. Find the next 3 terms of each sequence:

a. 1, 3, 9, 27, ...

b. 120, -60, 30, -15, ...

c. 5, -10, 20, -40

3. Determine whether each sequence is <u>arithmetic</u> or <u>geometric</u>:

a. 2, 4, 6, 8, ...

b. 2, 4, 8, 16, ...

c. 162, 54, 18, 6, ...

d. 98, 101, 104, 107, ...

*You can use the _____ _____ of a geometric sequence to write a _____ _____:

<u>Geometric Sequence</u>

8.6 Notes

4. Finding Terms of a Sequence:

Find the first, sixth, and twelfth terms of each sequence

a. $A(n) = 4 \cdot 3^{n-1}$

b. $A(n) = -2 \cdot 5^{n-1}$

Find the first, fifth, and tenth terms of the sequence:

$A(n) = -3(2)^{n-1}$

1 EXAMPLE Find the common ratio of each sequence.

a. $3, -15, 75, -375, \ldots$

$$3 \xrightarrow{\times(-5)} -15 \xrightarrow{\times(-5)} 75 \xrightarrow{\times(-5)} -375$$

The common ratio is -5.

b. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

$$3 \xrightarrow{\times\frac{1}{2}} \frac{3}{2} \xrightarrow{\times\frac{1}{2}} \frac{3}{4} \xrightarrow{\times\frac{1}{2}} \frac{3}{8}$$

The common ratio is $\frac{1}{2}$.

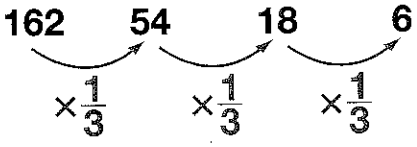
2 EXAMPLE Find the next three terms of the sequence $5, -10, 20, -40, \ldots$

$$5 \xrightarrow{\times(-2)} -10 \xrightarrow{\times(-2)} 20 \xrightarrow{\times(-2)} -40$$

The common ratio is -2. The next three terms are $-40(-2) = 80$, $80(-2) = -160$, and $-160(-2) = 320$.

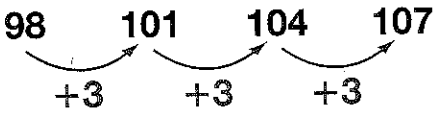
3 EXAMPLE Determine whether each sequence is arithmetic or geometric.

a. 162, 54, 18, 6, . . .



The sequence has a common ratio. The sequence is geometric.

b. 98, 101, 104, 107, . . .



The sequence has a common difference. The sequence is arithmetic.

4 EXAMPLE Find the first, fifth, and tenth terms of the sequence that has the rule $A(n) = -3(2)^{n-1}$.

first term: $A(1) = -3(2)^{1-1} = -3(2)^0 = -3(1) = -3$

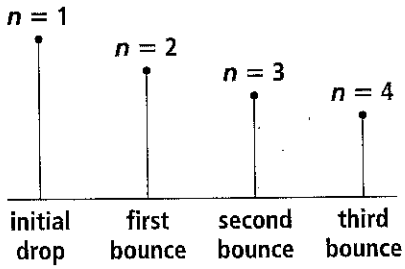
fifth term: $A(5) = -3(2)^{5-1} = -3(2)^4 = -3(16) = -48$

tenth term: $A(10) = -3(2)^{10-1} = -3(2)^9 = -3(512) = -1536$

5 EXAMPLE Suppose you drop a tennis ball from a height of 2 meters. On each bounce, the ball reaches a height that is 75% of its previous height. Write a rule for the height the ball reaches on each bounce. In centimeters, what height will the ball reach on its third bounce?

The first term is 2 meters, which is 200 cm.

Draw a diagram to help understand the problem.



The ball drops from an initial height, for which there is no bounce. The initial height is 200 cm, when $n = 1$. The third bounce is $n = 4$. The common ratio is 75%, or 0.75.

A rule for the sequence is $A(n) = 200 \cdot 0.75^{n-1}$.

$A(n) = 200 \cdot 0.75^{n-1}$ Use the sequence to find the height of the third bounce.

$A(4) = 200 \cdot 0.75^{4-1}$ Substitute 4 for n to find the height of the third bounce.

$= 200 \cdot 0.75^{3}$ Simplify exponents.

$= 200 \cdot 0.421875$ Evaluate powers.

$= 84.375$ Simplify.

The height of the third bounce is 84.375 cm.

Name _____ Class _____ Date _____

Practice 8-6

Geometric Sequences

Find the next three terms of each sequence.

1. $4, 12, 36, 108, \ldots$
2. $2, -8, 32, -128, \ldots$
3. $18, 9, \frac{9}{2}, \frac{9}{4}, \ldots$
4. $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$
5. $-2, 20, -200, 2000, \ldots$
6. $30, -10, \frac{10}{3}, -\frac{10}{9}, \ldots$
7. $\frac{1}{3}, 1\frac{1}{3}, 5\frac{1}{3}, 21\frac{1}{3}, \ldots$
8. $20, 4, \frac{4}{5}, \frac{4}{25}, \ldots$
9. $-100, -40, -16, -6.4, \ldots$
10. $40, 20, 10, 5, \ldots$

Determine whether each sequence is arithmetic or geometric.

11. $-8, -10, -12.5, -15.625, \ldots$
12. $5, 1, -3, -7, \ldots$
13. $1, \frac{2}{5}, \frac{4}{25}, \frac{8}{125}, \ldots$
14. $-0.2, -0.02, -0.002, -0.0002, \ldots$
15. $-10, -5, 0, 5, \ldots$
16. $6, -3, \frac{3}{2}, -\frac{3}{4}, \ldots$

Write a rule for each sequence.

17. $4, 12, 36, 108, \ldots$
18. $2, -8, 32, -128, \ldots$
19. $18, 9, \frac{9}{2}, \frac{9}{4}, \ldots$
20. $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$
21. $-2, 20, -200, 2000, \ldots$
22. $30, -10, \frac{10}{3}, -\frac{10}{9}, \ldots$
23. $1, 4, 16, 64, \ldots$
24. $6, 12, 24, 48, \ldots$
25. $125, 25, 5, 1, \ldots$
26. $50, 25, 12.5, 6.25, \ldots$

Find the first, fourth, and eighth terms of each sequence.

27. $A(n) = 2 \cdot 3^{n-1}$
28. $A(n) = 3 \cdot 4^{n-1}$
29. $A(n) = 3 \cdot 2^{n-1}$
30. $A(n) = -1 \cdot 5^{n-1}$
31. $A(n) = 4 \cdot 2^{n-1}$
32. $A(n) = \frac{1}{2} \cdot 2^{n-1}$
33. $A(n) = 0.1 \cdot 4^{n-1}$
34. $A(n) = -2.1 \cdot 3^{n-1}$
35. $A(n) = 10 \cdot 5^{n-1}$

Write a rule and find the given term in each geometric sequence described below.

36. What is the sixth term when the first term is 4 and the common ratio is 3?
37. What is the fifth term when the first term is -2 and the common ratio is $-\frac{1}{2}$?
38. What is the tenth term when the first term is 3 and the common ratio is -1.2?
39. What is the fourth term when the first term is 5 and the common ratio is 6?
40. Suppose a manufacturer invented a computer chip in 1978 that had a computational speed of s. The company improves its chips so that every 3 years, the chip doubles in speed. What would the chip's speed have been for the year 2002? Write your solution in terms of s.

Name _____ Class _____ Date _____

Reteaching 8-6

Geometric Sequences

OBJECTIVE: Finding the next terms of a geometric sequence

MATERIALS: None

- Multiplying a term in the sequence by a fixed number to find the next term forms a geometric sequence.
- The fixed number is called the common ratio.

Example

Find the next three terms of the sequence $3, -9, 27, -81, \ldots$

$3, -9, 27, -81, \ldots$

$-\frac{9}{3} = -3$

The common ratio is -3.

Note that each term in the given sequence is -3 times the previous term.

Let $A(n)$ = the value of the nth term in the sequence.

$A(5) = -3 \cdot -81 = 243$ ← **the common ratio times the fourth term**

$A(6) = -3 \cdot 243 = -729$ ← **the common ratio times the fifth term**

$A(7) = -3 \cdot -729 = 2187$ ← **the common ratio times the sixth term**

The next three terms in the sequence are $243, -729, 2187$.

Exercises

Find the next three terms in each of the following sequences.

1. $2, 8, 32, 128, \ldots$

2. $-3, 6, -12, 24, \ldots$

3. $1, -1, 1, -1, \ldots$

4. $12, 6, 3, \frac{3}{2}, \ldots$

5. $20, -10, 5, -\frac{5}{2}, \ldots$

6. $100, 10, 1, 0.1, \ldots$

7. $3, 15, 75, 375, \ldots$

8. $-8, -12, -18, -27, \ldots$

9. $1.5, 4.5, 13.5, 40.5, \ldots$

10. $8, -\frac{8}{3}, \frac{8}{9}, -\frac{8}{27}, \ldots$

11. $7, -14, 28, -56, \ldots$

12. $100, 50, 25, 12.5, \ldots$

13. $8, 32, 128, 512, \ldots$

14. $76, -38, 19, -9.5, \ldots$